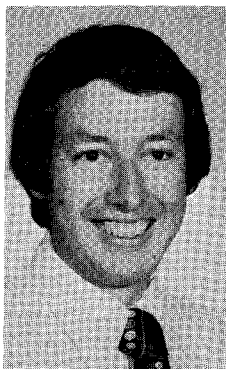


Moment Magnification Tests of Prestressed Concrete Columns



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A T-shaped prestressed concrete column was designed to behave in an unstable condition with the object of testing a mathematical model previously used for making design recommendations. Tests of five specimens are reported.

Design recommendations are suggested for modifying certain provisions of the ACI Code in regard to prestressed flanged sections. Although the observed moment-curvature relation did not agree well with the computed values, the overall column behavior was predicted accurately. The model is felt to be validated.

In previous papers^{1,2} a mathematical model of the prestressed concrete non-rectangular column was presented and used for investigating slenderness effects.

On the basis of these studies, it was recommended that certain modifications² be made in the moment-magnifier procedures of the ACI Building Code,³ to make them more applicable to prestressed flanged sections. These suggestions were adopted in the Recommended Practice for the Design of Prestressed Concrete Columns and Bearing Walls.⁴

Further, it was urged that design curves should be prepared by manufacturers, using such mathematical models, for standard sections. Use of design curves of this type represents the "rational analysis" which is, in fact, preferred by the ACI Building Code.

The analytical procedure used in these studies was based on well-established principles, but on very limited experimental evidence. A few previous investigators had conducted tests of simple cases which could be used to confirm predicted behavior, but it was felt that further experimental confirmation was desirable. The intent of this paper is to report on an experimental program conducted for this purpose.

Mathematical Model

The analysis is embodied in a conceptually simple but lengthy computer program which depends on the assumptions that:

1. Plane sections remain plane.
2. The material laws are known and are not path or time dependent. (Such dependencies could also be included but are not in the program presently in use. Creep has been simulated by lengthening the strain scale.) Any uniaxial stress-strain relation can be

used; data can be entered in functional form or a curve will be fitted to pairs of values from an experimental relation.

3. Curvature can be approximated by the second derivative of displacement; in concrete members material failure will generally occur long before this assumption is invalidated.

4. Boundary conditions are known. Again, any boundary conditions can be accommodated.

Once these assumptions have been made, the problem is solved by numerical integration; computational errors can be kept well within limits required for practical problems.

Design of Tests

Fig. 1 shows the load-moment relations for a column with equal end

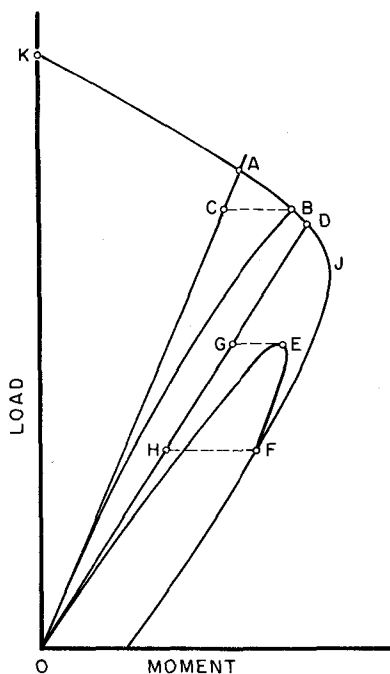


Fig. 1. Load-moment relations showing stable and unstable load paths.

moments tending to cause single curvature. The outer curve is the usual interaction curve of axial load versus ultimate moment. Path OA represents the relation between load and end moment as load is increased at constant eccentricity.

Due to the curvature of the member and the consequent moment magnification, the mid-height moment is greater than the end moment, and follows Path OB. At Point B the mid-height moment causes failure of the member, which occurs when the end conditions are represented by Point C.

If the load is applied at a greater eccentricity, the load-end moment relation might follow Path OD. In that case the mid-height moment, accord-

ing to the theoretical model, follows Path OEF; at Point E the lateral deflection begins to increase rapidly, and the column becomes unstable, being unable to sustain the load. If the load is reduced (or if vertical displacements are controlled rather than loads) the mid-height moment follows Path EF to failure at Point F, while the end conditions retreat from Point G to H.

Thus, the behavior of a member loaded along Path OD is quite complex, and if the mathematical model should be proved correct for these conditions, it can be said to have been quite severely tested and can be applied with some confidence in other cases.

The unstable response described

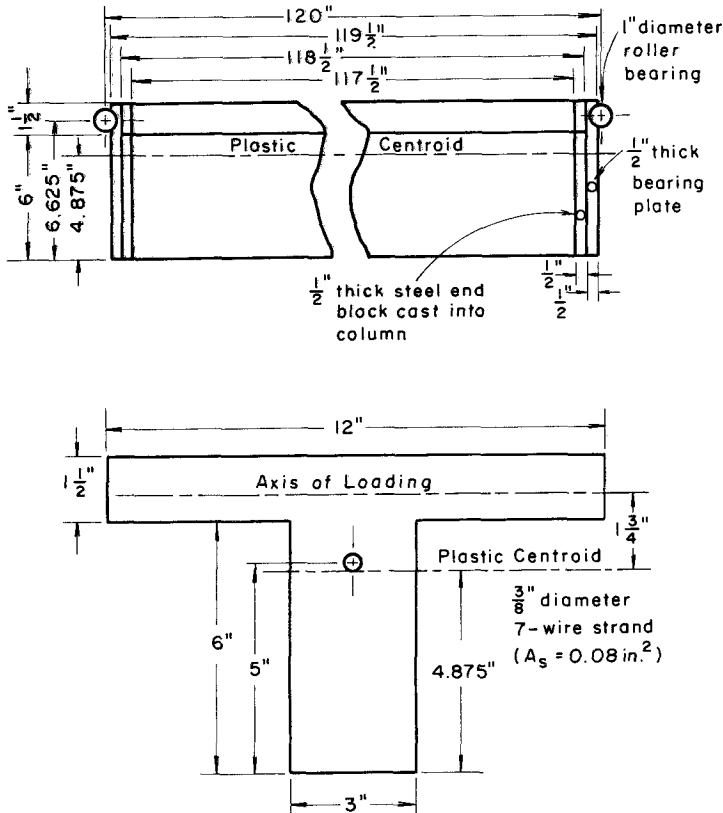


Fig. 2. Specimen details.

above occurs when the eccentricity is such that the load does not reach the "balanced" value (Point J on Fig. 1); and the balanced load as a ratio of the maximum pure axial load (Point K on Fig. 1) is increased by the presence of a compression flange and of prestressing. Therefore, it was decided to test prestressed T sections loaded in such a way that the unstable behavior of Path OEF could be expected.

Details of Test Specimens

Six similar columns were prepared for testing; one was used to try out and develop the instrumentation, and the results reported herein refer to the remaining five.

Specimens were 10-ft long T sections with 12-in. wide by 1½-in. deep flanges, 7½-in. overall depth, and 3-in. wide stems. Details are shown in Figs. 2, 3, and 4.

The columns were pretensioned with a single ¾-in. diameter seven-wire strand, with the stress-strain diagram shown in Fig. 5. The bond anchorage zone was reinforced as shown in Fig. 3 with steel having a yield point of 60 ksi.

Standard 6-in. diameter by 12-in. cylinders were cast simultaneously with the columns and tested just before or after the column tests. (The same strain-controlled machine was required, and cylinder tests were generally within a day of the column tests.)

The first column was tested 57 days and the last 123 days after casting. The stress-strain curves obtained from the concrete cylinders are shown in Fig. 6. It may be noted that the strains at peak stress were somewhat higher than might have been expected, but this cannot be explained by the authors.

During manufacture, the prestress

was controlled by measuring the elongation; the initial value was between 15.13 and 15.95 kips for different columns, and losses were estimated at 2.53 to 2.60 kips. According to the mathematical model, the column behavior was not especially sensitive to the prestress force (see Fig. 7), and therefore no attempt was made to measure prestress loss.

Columns were cast with a ½-in. steel plate welded to the end block reinforcement as shown in Fig. 3. A second ½-in. steel bearing plate with machined groove was subsequently

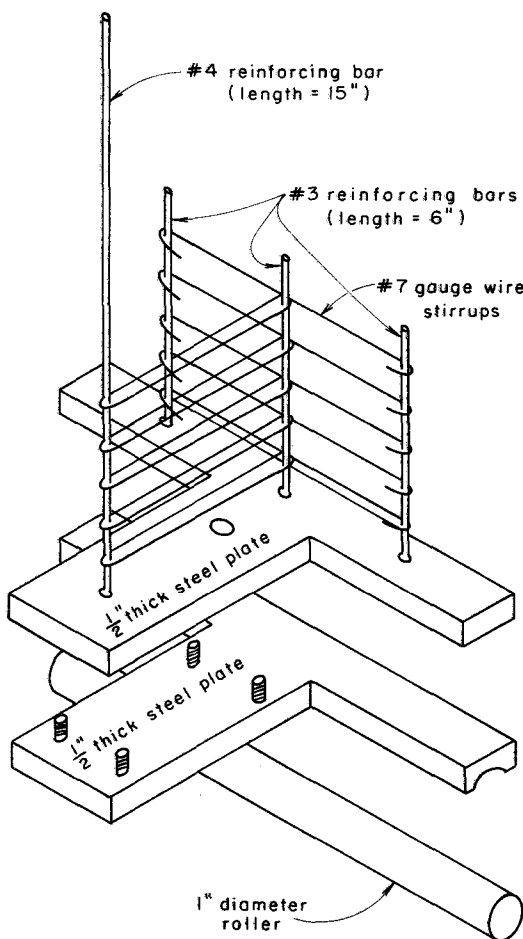


Fig. 3. End block and loading arrangement.

screwed in place, also as shown in Fig. 3.

A 1-in. diameter roller, greased with a graphite lubricant, was placed in this groove, and received by a similar groove in the bearing blocks shown in Fig. 4. These bearing blocks were fitted with retainer angles, to keep the column end from slipping out at failure. Thus, the thrust was applied as a line load at the desired eccentricity.

Instrumentation

To measure lateral deflection of the column, a linear variable differential transformer with a range of ± 1 in. was

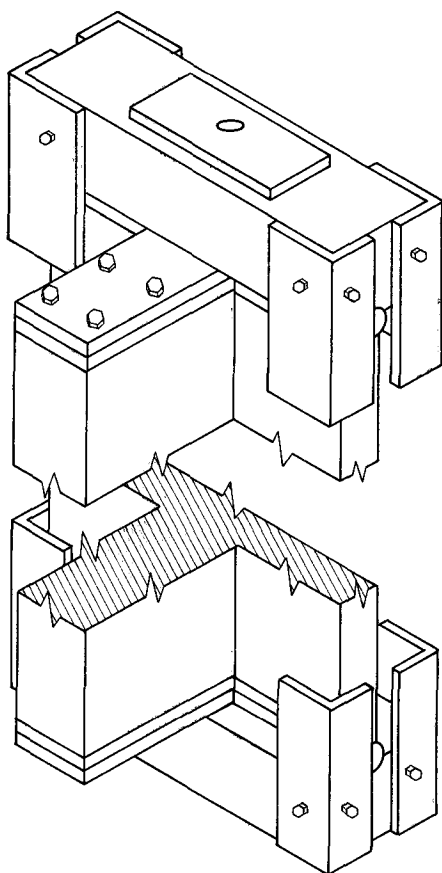


Fig. 4. Assembled loading blocks.

mounted on a small carriage which could be driven up and down a reference track mounted beside the column. The tip of the transformer core was fitted with a teflon runner bearing against a strip of masking tape on the column surface. The output voltage from this device was fed to an x-y plotter, which could plot the deflected shape of the column with an accuracy of about ± 0.005 in. in 3 seconds.

To measure the curvature of the column, a series of 7½-in. long cantilevers were mounted in a straight line down one side of the web, parallel to the column axis (see Fig. 8). These aluminum cantilevers were fixed to brackets epoxied to the concrete surface, and the tip deflection of each was measured by a linear variable differential transformer mounted on the next bracket. This deflection represented the departure of the column curve from its tangent in the 7½-in. length.

Then it is easily shown that the column curvature is $\Phi = 2\Delta / l^2$, where Δ is the cantilever tip deflection and l is the length of the cantilever.

These deflections could be read to an accuracy of 0.000013 in.

Vertical displacement due to axial shortening and bowing of the column was also read by means of an LVDT and this reading, together with the load reading from the Tinius Olsen testing machine, was fed into a second x-y recorder to produce a continuous record of load versus displacement.

In addition to the two x-y recorders, output voltages were scanned by a Vidar digital voltmeter, scanner, and recording system. Data included:

- (a) Clock time (day, hour, minute, second).
- (b) Output voltage from load measuring transducer.
- (c) Seven output voltages from cantilever tip deflection measurements.

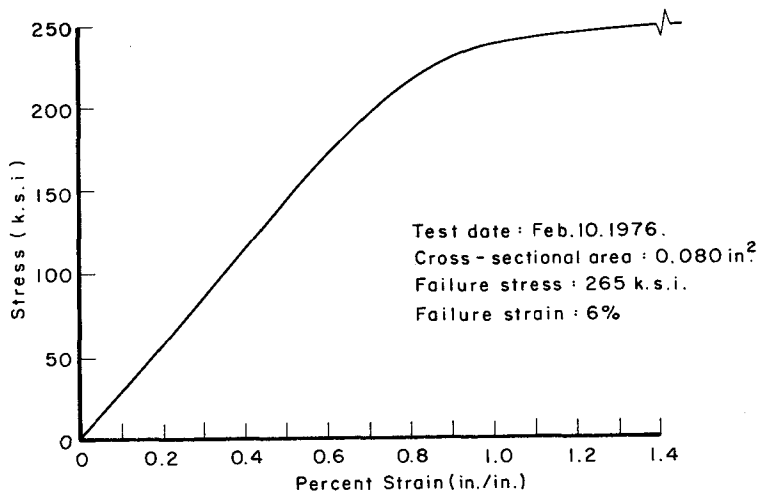


Fig. 5. Stress-strain curve of prestressing strand.

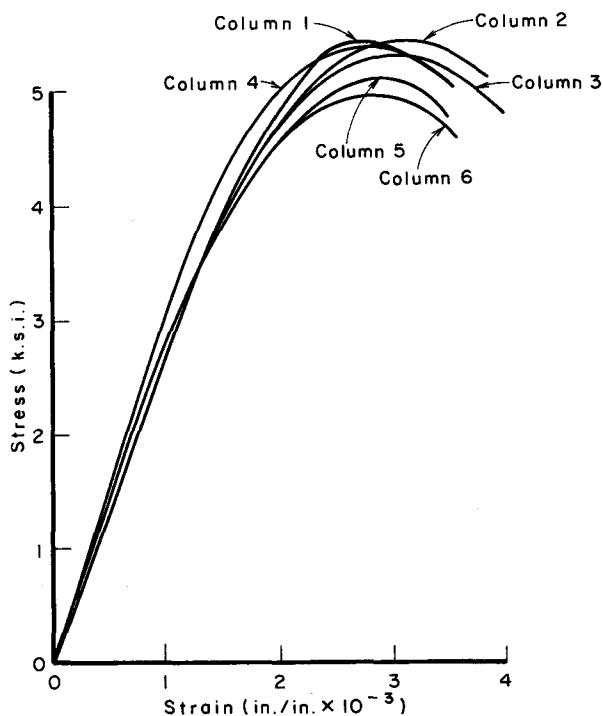


Fig. 6. Stress-strain curves from concrete cylinder tests.

- (d) Output voltage from the axial deflection measuring transducer.
- (e) Input voltage to each transducer.

Voltages were recorded on magnetic tape for computer processing; a digital PDP11 mini-computer connected to the Vidar recording system could be used to give curvatures, deflections, and so forth while a test was in progress.

Loading

A Tinius Olsen strain-controlled testing machine was used. The load-deflection curve for the testing machine itself was previously determined by jacking the platens apart; it can be shown that if this curve was flatter than the unloading curve of the column, instability of the machine-specimen system would occur. It was

concluded that the descending branch of the column loading curve could be satisfactorily obtained.

In testing, all electrical equipment was switched on 4 hours in advance to ensure stationary voltage outputs. A small load (typically 5 kips) was applied and removed, to ensure correct seating of the end bearings.

Vertical deflection was imposed on the specimen at a rate of 0.0025 to 0.0050 in. per minute, and instrument readings were taken at load intervals of 5 kips up to 35 kips; thereafter, readings were taken at 2-kip intervals and then as often as possible when the descending branch was reached. A typical test took about 1 hour.

Test Results

In each column, fine hairline cracks spaced from 8 to 12 in. apart appeared in the central region as the peak load

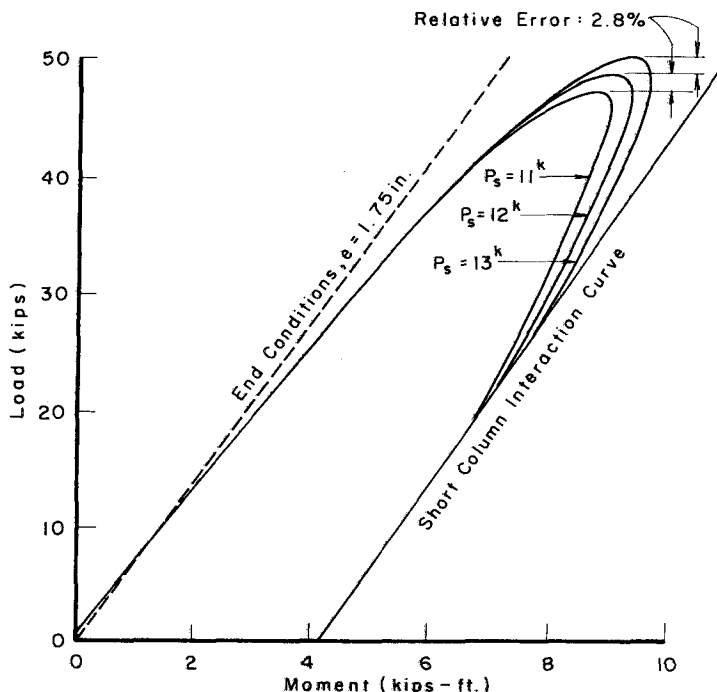


Fig. 7. Calculated effect of prestress losses on peak load.

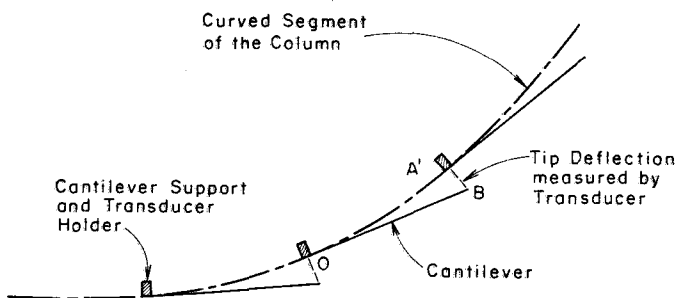


Fig. 8. Devices for measuring curvature.

was approached within 1 kip. With increasing lateral deflections, the cracks spread along the column and grew inward toward the column axis; about 3 in. from the outer fiber of the web, they usually branched diagonally to a Y-shape. Eventually one of these cracks widened, and the concrete in the flange opposite the crack failed in compression (see Fig. 9).

Moment-curvature relation

Curvatures were computed, as described above, from the recorded tip deflections of the cantilever devices. The deflection curves obtained from the carriage-mounted LVDT were also used to provide curvature values in the following manner: several points from the deflection curve were fed into a computer which then fitted a smooth curve and differentiated it twice.

This procedure was used, rather than numerical differentiation of the observed deflected shape, because the latter was not obtained with sufficient accuracy. In the method described, the results are, in effect, "averaged out" before differentiation. Moments were taken as the load times the deflection measured from the thrust line.

The results from both these methods were disappointing. As shown on Fig. 10, they indicated consistently smaller curvatures than were

calculated from the measured material properties. Several possible causes for this discrepancy were considered: initial imperfections in column shape, errors in concrete strain measurements, variations in prestress, differences between concrete strains in cylinders and columns; but each was found either to have negligible effect or to make the discrepancy worse.

Thus, no satisfactory explanation can be advanced, except that the instrumentation was simply not accurate enough. Each plotted point in Fig. 10 is the mean of several values from one

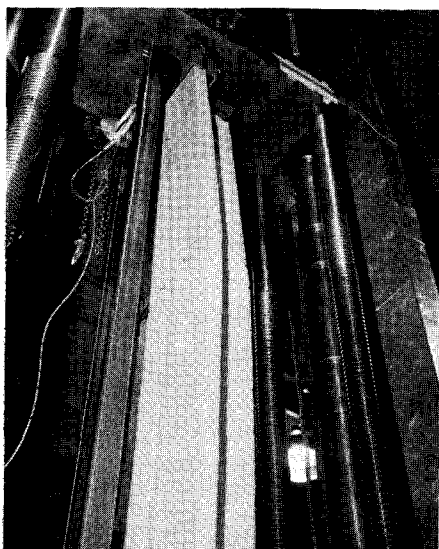


Fig. 9. Failed specimen in testing machine.

column at one load, and the spread of these values was quite large, averaging about $1.5 \times 10^{-5} \text{ in.}^{-1}$ on the curvature scale, with a larger spread at higher moments. The peak loads did indicate that the columns were a little stiffer than was calculated.

Load-moment relation

The essential purpose of the program was to test the ability of the mathematical model to predict the load versus mid-height moment relation. This implies the moment magnification factor during the ascending and descending branches of the equilibrium curve, before and after the point of instability. The results of

this exercise are shown on Figs. 11 to 15.

It will be seen that the form of the curve was very well predicted. The peak load was underestimated somewhat in each case, by 8.1, 10.1, 1.7, 7.1, and 5.5 percent in Columns 2 to 6, respectively.

The adjustment in prestress necessary to bring the predicted and measured peak loads into agreement is indicated on Figs. 11 to 15, and, in some cases, the discrepancies could be accounted for by an incorrect estimate of losses; in other cases, the differences are too large for this cause alone.

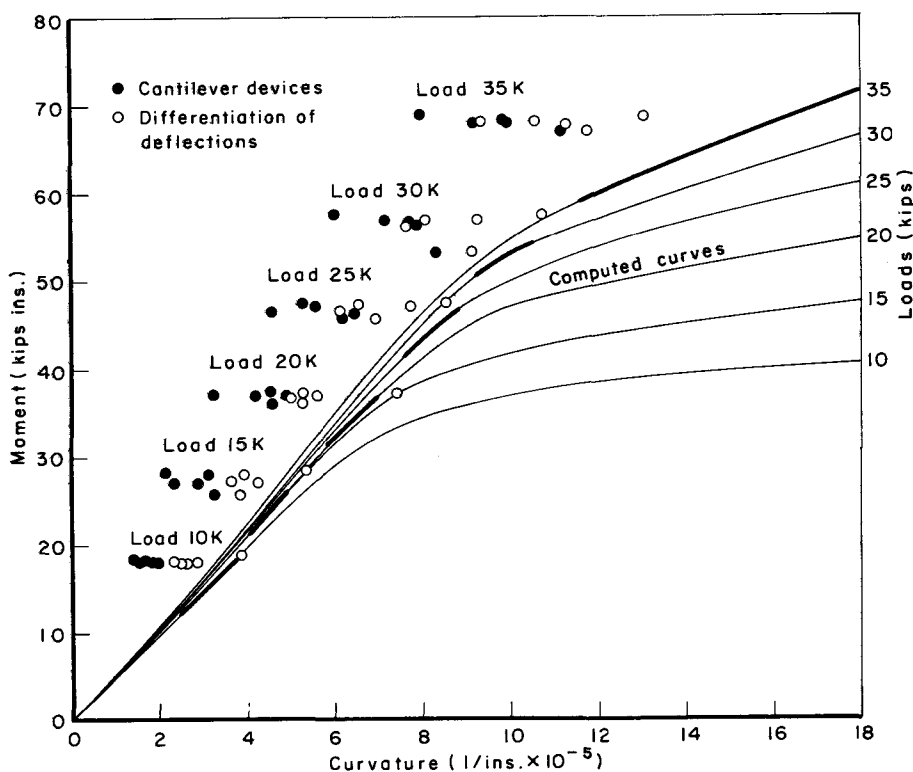


Fig. 10. Measured and computed curvatures.

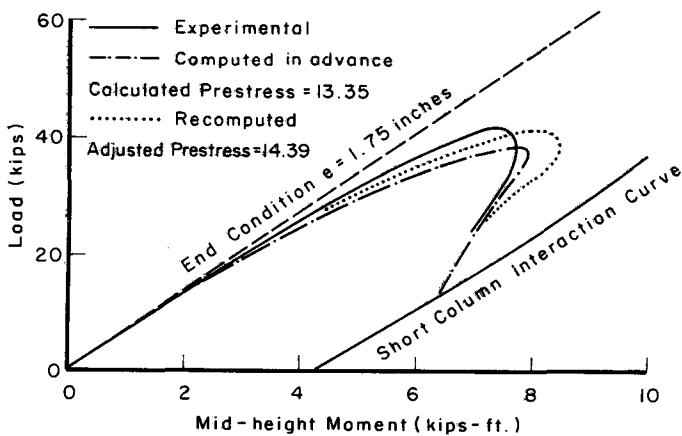


Fig. 11. Observed and predicted load paths for Column 2.

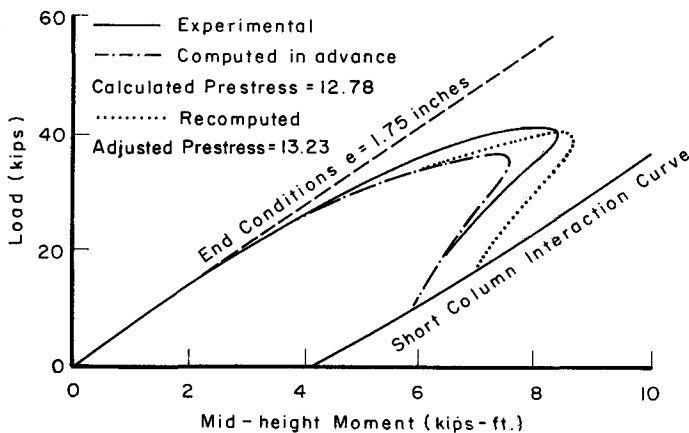


Fig. 12. Observed and predicted load paths for Column 3.

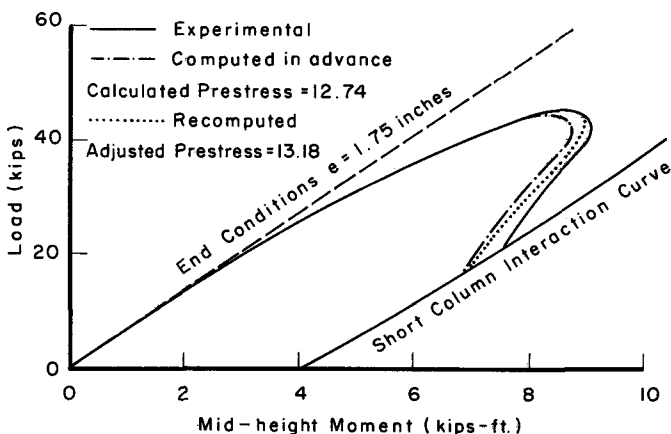


Fig. 13. Observed and predicted load paths for Column 4.

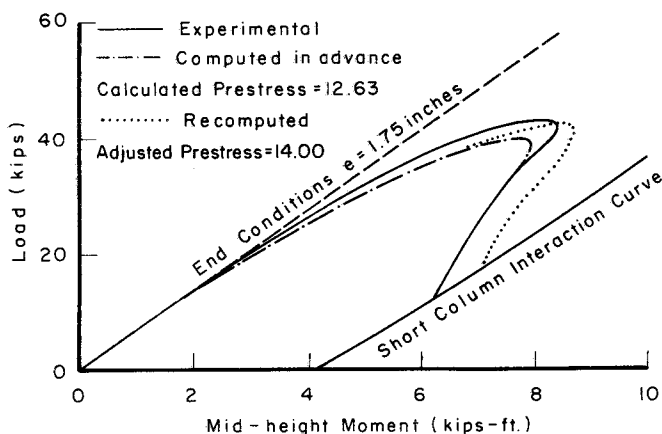


Fig. 14. Observed and predicted load paths for Column 5.

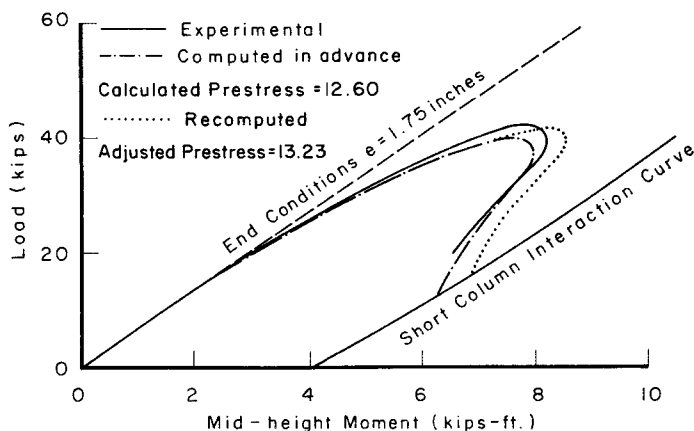


Fig. 15. Observed and predicted load paths for Column 6.

Summary and Conclusions

A mathematical model had previously been used² to predict the behavior of prestressed load-bearing wall panels and columns, and recommendations for design procedures had been made on the basis of these predictions. It was desired to gain further experimental evidence of the accuracy of this model.

A column with simple boundary

conditions was designed to behave in an unstable manner with a descending branch of the equilibrium curve; tests of five specimens have been reported here.

Both ascending and descending branches of the equilibrium curve, as well as the peak load, were predicted with sufficient accuracy to give confidence in the mathematical model; but the moment-curvature relation, which is acquired at an intermediate stage of the calculations, did not agree well with observed values. No convincing

reason could be found for this latter discrepancy other than insufficient accuracy in the instrumentation.

Design Recommendations

It has been proposed¹ that the type of analysis which has been investigated here should be used for members which fall outside the limits covered by code formulas; it may also be used to produce design curves for standard units.

In the development of empirical design procedures, too, it is not necessary to perform laboratory experiments in every case when reasonably accurate assessments of member behavior can be obtained by means of a computer. In fact, this procedure was followed² in recommending certain modifications to the ACI Code for application to prestressed flanged sections:

1. That ACI 318-71 Eqs. (10-7) and (10-8) be replaced by:

$$EI = E_c I_g / \lambda$$

where

$$\lambda = 15 - 25 (P_u / P_o) \geq 2.5$$

When there is a compression flange:

$$\lambda = 17.5 - 37.5 (P_u / P_o) \geq 2.5$$

When there is no compression flange:

P_o = the axial load capacity at zero eccentricity.

2. That ACI 318-71 Eq. (10-9) be replaced by:

$$C_m = 0.7 + 0.3 (M_1 / M_2)$$

3. That effective sections as defined in Reference 4 be used in slenderness calculations.

Further studies of this kind are in progress, supported by the experimental evidence reported herein.

References

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4. PCI Committee on Prestressed Concrete Columns, "Recommended Practice for the Design of Prestressed Concrete Columns and Bearing Walls," PCI JOURNAL, V. 21, No. 6, Nov.-Dec. 1976, pp. 16-45.

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Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by January 1, 1978.